Efficiency of Direct-Radiator Loudspeakers

VINCENT SALMON*

An analysis of the factors affecting the efficiency of the type of speaker most commonly employed in sound reproduction installations.

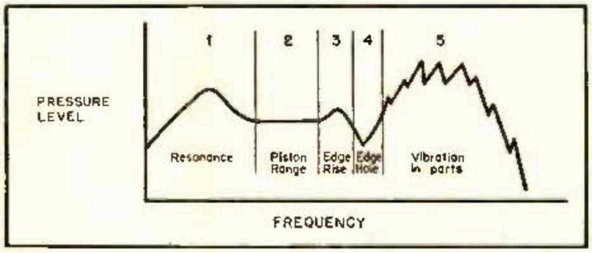

Fig. 1. Characteristic frequency regions in the free-space axial sound pressure response of mass-controlled direct-radiator speaker in infinite baffle.

THE MOST WIDELY USED loudspeaker is the direct-radiator, moving-coil, mass-controlled type. In this note are discussed some of the factors influencing its available-power efficiency, and especially that in the piston-range.

In any speaker the response is commonly understood to be the acoustic output as a function of frequency for some constant electrical input. On the other hand, the efficiency is usually a single value, a sort of figure of merit, so that the speaker may be compared with others. As may be expected, there are many types of efficiencies, depending on which combination of electrical input, acoustical output, or test signal is used.

For the purposes of this discussion, we shall consider the efficiency that is related to the program energy output of a speaker used indoors. Thus the measure of acoustic output is the total radiated power, and especially that near 400 cps, where the program energy peaks. Consider next where this frequency region lies with respect to those observed on a pressure-response curve.

If the free-space axial sound pressure response of a direct-radiator speaker in an infinite baffle (chosen because it is a reproducible mounting) is taken as a function of frequency, it is found that there are at least five regions of interest, starting at the low-frequency end. As shown considerably smoothed in *Fig.* 1, the first is centered about the fundamental resonance of the speaker. At very low frequencies the response rises 12 db/octave, shows a peak at resonance (for the usual speaker), and then falls off. In region 2 the response is practically constant; this is the piston range where we shall calculate the efficiency. The program energy spectrum usually maximizes in this range. In region 3 the response peaks at the edge rise, where the cone is about a quarter wave long. Waves in the cone material are reflected from the outer termination and arrive back at the voice coil so as to lower the mechanical driving-point impedance, thus increasing the velocity and hence the energy radiated. With increasing frequency, the half-wave condition is next approached, and in region 4 the resulting edge hole is seen, arising from the high impedance reflected into the voice coil from the cone edge. Beyond this region the wave pattern in the cone becomes more complicated, the driving-point impedance fluctuates considerably, and the speaker becomes more directional. All of these add up to the ragged, bulged response of region 5, that of cone vibration in parts. Only in region 2 is there anything approaching a level response; in addition, the program energy the speaker must handle is greatest in this range, about 50 per cent lying between 200 and 600 cps.

In this region a number of phenomena cooperate nicely to simplify the analytical treatment. First, the cone is moving practically as a rigid piston in an infinite baffle; hence the Rayleigh radiation impedance may be used. Also in the piston range the mechanical driving-point impedance is principally that of a mass, since the speaker is working well above the fundamental resonance and well below vibration in parts. Third, the speaker is not yet too directional, so that the sound pressure varies little with angle, permitting the total radiation to be calculated quite simply from the sound pressure. Fourth, for most speakers the frequency is still moderate so that the (series) radiation resistance varies as the square of the frequency. And finally, in this frequency range the inductance of the voice coil may be neglected. All this makes possible a simple expression for the piston-range efficiency.

Efficiency is expressed as a ratio of acoustic output to electrical input. To be useful, the measure of acoustic output should correspond to what the ear hears. Since for indoor use the total acoustic power radiated plus room effects determines the sound pressure at the ear, we choose power radiated rather than the axial sound pressure (which serves for outdoor listening). As a measure of electrical input the most suitable for the present purpose is that power which the representative test amplifier used will deliver to its rated load. Note that this is not the power to a matched load (one equal to source impedance), since amplifiers are not operated that way. This rated load is simply that given by the number on the output terminals: if they are marked 16 ohms and the amplifier is adjusted for one watt into a 16-ohm load, the power available *to the speaker* (*not* power available *from the source*) is said to be one watt. It is necessary to use this criterion, since the maximum power from the amplifier is of no use to us: it is the maximum "undistorted" power delivered to the optimum load that is useful. The speaker efficiency on this basis is then the number which when multiplied by the power available to the speaker gives (as closely as possible) the acoustic power from the speaker, under the infinite-baffle condition. In the frequency range considered, a cabinet will give a somewhat rougher response, fluctuating about that of the baffle.

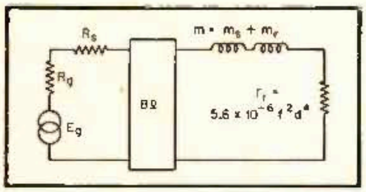

Fig. 2. Equivalent piston-range circuit of speaker with amplifier connected. See text for symbols.

Electromechanical Circuit

In the frequency range considered, usually 200 to 600 cps for 8- to 15-in. speakers, the output (source) impedance of most satisfactory amplifiers may be considered to be a constant resistance. With this assumption, plus the others obtaining in the piston range, the equivalent electromechanical circuit will be as in *Fig.* 2, where the force-voltage analogy is used. Here E_g and R_g are the source open-circuit voltage and impedance, respectively; R_s is the d.c.

* *Stanford Research Institute, Stanford, California.*

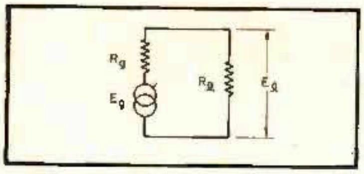

Fig. 3. Measurement of power available to amplifier load.

resistance of the speaker in ohms; Bl is the product of flux density in gauss and conductor length in cm; m is the total moving mass, including the speaker mass m_s and the radiation mass $m_r = 6.6 \times 10^{-2} d^3$ (m_r is in grams, d is effective cone diameter in inches); and the series radiation resistance is given by $r_r = 5.6 \times 10^{-6} f^2 d^4$ in mechanical ohms, where f is the frequency in cps. If the r.m.s. cone velocity is v (in cm./sec.), then the radiated acoustic power in watts is $P_a = 10^{-7} v^2 r_r$.

The power available to the load is obtained by replacing the speaker by R_l, the rated load impedance of the amplifier, as in *Fig.* 3. It is easily shown that the power in the load is

$$P_l = \frac{E_g^2}{R_l} \cdot \frac{1}{\left(1 + \frac{R_g}{R_l}\right)^2}. \qquad (1)$$

To digress a moment, note that E_g^2/R_l is the load power if the source impedance R_g were zero; hence the second factor on the right expresses the deviation from this condition caused by finite source impedance, and hence is also a measure of the damping effect of the amplifier on a speaker load. The term *regulation* has been given to the quantity

$$D = 20 \log_{10} \left(1 + \frac{R_g}{R_l}\right). \qquad (2)$$

For properly loaded push-pull 6L6's, the regulation is about 15.5 db, while a pair of 6B4's will have a regulation near 3.5 db. If sufficient feedback is used, D can be reduced to as little as 1.0 db, although 3 to 5 db is nearer average commercial practice in high-quality sound system amplifiers. To measure D, simply load the amplifier properly and read the output voltage. Then remove the load; D is the db rise in output voltage. In the so-called constant-voltage audio distribution system, D is 3 db or less.

To return to the main topic, it is seen that the efficiency is given by

$$\eta = \frac{P_a}{P_l} = \frac{10^{-7} v^2 r_r \, R_l \left(1 + \frac{R_g}{R_l}\right)^2}{E_g^2}. \qquad (3)$$

By calculating the cone velocity to be expected when the connections of *Fig.* 2 are used, the following expression finally results:

$$\eta = \left[\frac{R_s}{R_l}\left(\frac{R_g + R_l}{R_g + R_s + \frac{10^{-9}(Bl)^2}{(r_r^2 + 4\pi^2 f^2 m^2)^{\frac{1}{2}}}}\right)^2\right] \times \left[\frac{(Bl)^2}{R_s}\right]\left[\frac{r_r}{(r_r^2 + 4\pi^2 f^2 m^2)}\right] \times 10^{-9}. \qquad (4)$$

Now we can begin to introduce the piston-range approximations. In practically all direct-radiator speakers, the last denominator term in the first brackets can be neglected, because of the low efficiency. Since the speaker is mass controlled, the first denominator term in the last brackets can be neglected, and since the frequency is not too high, $r_r = 5.6 \times 10^{-6} f^2 d^4$. For most properly used direct-radiator speakers R_s/R_l is near 0.85. Finally, let us examine the

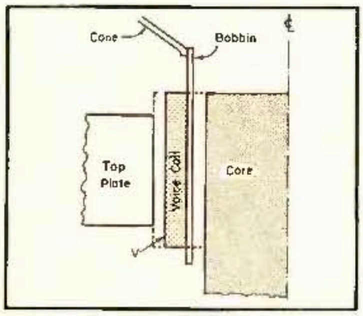

Fig. 4. Geometry of voice coil and gap volume.

term in the second brackets at length.

Since this term contains much of the information which determines the efficiency, its algebraic manipulation will be described in some detail. Resistance may be expressed in terms of conductor length l and cross section area A by $R = \rho l/A$, where ρ is the resistivity. Thus $l^2/R = l^2 A/\rho l = lA/\rho$. But lA is the volume of the conductor. Set this volume equal to a fraction α of the total gap volume V, which is defined in *Fig.* 4 to include that in the voice coil overhang. The fracton α is the space factor, and is a measure of how efficiently the voice coil makes use of the magnetic flux density available. Now suppose that the magnetic energy W in the gap is determined by the measured flux density, using the relation $W = B^2 V/8\pi$. Then $B^2 = 8\pi W/V$. Thus after all substitutions $B^2 l^2/R_s$ becomes $8\pi\alpha W/\rho$, where W is the gap magnetic energy in ergs and ρ is the voice coil resistivity in ohm-cm.

If all the piston-range substitutions and approximations are made in the efficiency formula, there results

$$\eta \doteq \left[\frac{\frac{2.44 \times 10^{-9}}{0.3}}{\left(1 - \frac{R_g}{1 + \frac{R_g}{R_l}}\right)}\right] \frac{C\alpha W}{(m/d^2)^2}.$$

(ergs, grams, inches) *(5)*

Here C is the ratio of the conductivity of the voice coil to that of copper. The coefficient in brackets has the following values for these output stages: PP6L6, no feedback, about 2.6×10^{-9}; PP6L6, considerable feedback, (6 db regulation) 2.9×10^{-9}; and PP6B4, 3.1×10^{-9}. To repeat, α is the space factor, the fraction of the gap volume used by the voice coil; W is the magnetic gap energy in ergs; m is the total effective moving mass in grams; and d is the effective cone diameter in inches. A good approximation to d is the distance from the top of the innermost bead on the annulus to the opposite outer clamp diameter of the cone. See *Fig.* 5.

Analysis of Formula

Let us now examine this expression more closely. The effect of frequency has canceled out because of the two main assumptions that the principal mechanical-impedance component is that of the moving mass, and that the radiation resistance varies as the square of the frequency. Two important factors in the efficiency are seen to be the space factor and gap energy. In the form given, the influence of magnet size is easily seen, for magnets are characterized by a guaranteed maximum externally available energy. In a well designed PM speaker, about 40 per cent of this energy apears in the useful gap, the remainder being lost in leakage and fringing. Thus efficiency may be increased by using larger magnets only as long as iron saturation is not a

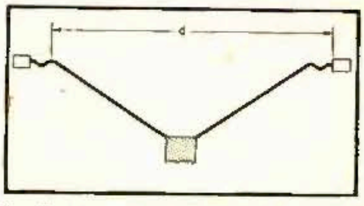

Fig. 5. Measurement of effective cone diameter d.

limiting factor. Hence the gap-energy term in the efficiency reflects the economics of speaker manufacture, and aids in comparing speakers; neither the flux density nor the total flux will permit direct comparison. The use of Alnico V in place of Alnico III is an example of this: while the cost per pound of V is greater, it has over three times the energy per pound, so that the cost per million ergs is less, offsetting even some of the mechanical problems of fastening Alnico V magnets in place. It is magnetic energy the customer pays for, not gauss or maxwells.

The space factor α can be improved by using a rectangular or ribbon shape instead of circular for the voice-coil conductor. However, if a reasonably high impedance is needed, the ribbon must be made quite thin, and then the inter-turn insulation may be a very appreciable fraction of the total thickness. For ordinary direct-radiator speakers, the space factor—which includes allowances for clearances as well as insulation and precision of wire lay (see *Fig.* 4)—may be about 20 per cent. Voice coils of edgewound ribbon sometimes reach 30 per cent by also omitting the bobbin, but one sturdy enough for use in a "woofer" should offer only a slight advantage. The theoretical advantage should be that of the square over the circle, or 27 per cent gain. The actual gain cannot be very great, for the coil-to-pole-piece clearances required in low-frequency

[*Continued on page 42*]

SPEAKER EFFICIENCY

[from page 14]

speakers are fairly large (10 to 15 mils) and are primarily responsible for the space factor being so low. These facts, combined with the greater manufacturing difficulties and costs of edgewound coils, account for the more widespread use of round-wire voice coils.

The product αW is the magnetic energy embraced by the actual conductor in the voice coil. It can be determined for a completely assembled speaker by measuring the blocked force produced by d.c. power in the voice coil. The use of d.c. is permissible because of the piston range characteristics enumerated earlier. The force F is $Bl I_s$, and the power P is $I_s^2 R_s$. Thus the quotient $F^2/P = (Bl)^2/R_s$, which by the relation developed for gap energy is also $8\pi\alpha W/\rho$. Thus

$$\alpha W = \frac{\rho}{8\pi}\frac{F^2}{P} \qquad (6)$$

If the force is measured in pounds and the power in watts, then $\alpha W = 1.35 \times 10^5 F^2/PC$ ergs. If a spring scale is used for measuring the force, the reading must be taken with the voice coil restored to and in its undeflected rest position so as to avoid the undesired extra contribution from the restoring force of its annulus and centering member. Special test devices are easily made for measuring or comparing the product αW.

Voice-Coil Winding

If the voice coil is wound with other than copper, but with the same dimensions and space factor, then the relative conductivity will affect the efficiency by the factor C. If aluminum is the alternate material to copper, its C is 0.64, thus lowering the efficiency. However, its density is only 0.33 that of copper, so a smaller total mass should result, increasing the efficiency by the $1/m^2$ factor. It turns out that a change from copper to aluminum will not improve the efficiency unless the copper voice-coil mass is over 30 per cent of the total moving mass. In most cases this condition will obtain only when the speaker is specifically designed as the low-frequency end of a two-way system.

The factor (m/d^2) in the denominator is a measure of the surface density of the moving system. It varies (in units of grams and inches) from about 0.15 to 0.30 for standard speakers in the 8- to 18-in. range. In the popular 10- and 12-in. sizes it is about 0.25. Thus there is not too much variation in this factor, which leaves the gap energy as the principal variable in the efficiency. The value of m may be estimated by first measuring f_o, the frequency of resonance. Then add to the voice coil region of the cone of the speaker a known non-metallic mass m_1, such as modeling clay, which will cling to and vibrate with the cone. Measure the new resonant frequency f_1. The mass m is then given by $m = m_1/[(f_o/f_1)^2 - 1]$. It is best to make m_1 fairly close to m.

In the piston range the frequency is usually low enough so that the sound pressure will not vary much with angle. On this basis a speaker which would absorb and convert into the acoustical form all power available to it would produce at a distance r feet in front of it a sound pressure level given by

$$L_p = L_w - 20\log_{10} r + 92.5 \qquad (7)$$

Here all the radiation is assumed to be confined to a hemisphere in front of the speaker and infinite baffle. L_p is the sound pressure level in decibels above 0.0002 dyne/cm²; the unit is the dbp, from analogy with *dbm*. L_w is the input available power level in decibels above one milliwatt, and thus is in *dbm*. A non-directional speaker of 100 per cent efficiency should thus produce, with one watt available input, an axial sound pressure level at ten feet of 102.5 dbp. A standard 12-in. speaker will have a piston-range efficiency of about 2 per cent, so it will produce a level of 85.5 dbp under the above conditions. This will be increased by directional effects starting near the end of the piston range. This use of free-space measurements of sound pressure for calculating the total radiation in a room is justified in the piston range, where the effect of the room is small.

It is to be noted that this simple use of efficiency in pressure level calculations is not possible if the efficiency is based on the ratio of motional to total impedance. The efficiency should reflect two characteristics of the speaker: the ability to absorb power from the load without causing undue distortion; and the ability to convert the power absorbed into the acoustical form. The "energy efficiency" calculated by motional resistance measurements neglects the first factor.

When speakers are audited properly, they may be rank ordered in terms of loudness. It will be found that this order is not necessarily that of the piston-range efficiencies. This arises from the determining influence of the frequency range from 1000 to 4000 cps on the loudness. Since these frequencies are outside the piston range, no correlation is, of course, to be expected. However, for speakers of similar total radiation response shape the piston-range efficiency should be a very good indication of the loudness rating.

In closing, it is believed that the efficiency relations discussed provide a useful means of comparing the basic performance of direct-radiator speakers. The factors entering into the expression may be measured independently, or the efficiency may be calculated from observed sound pressure levels. The principal factors are shown to be the gap energy and space factor, and their relation to some design considerations has been noted.